Mining Frequent Items in a Stream using Flexible Windows

Toon Calders, *Eindhoven Technical University*

Nele Dexters, Bart Goethals, *University of Antwerp*

DBDBD 2006 – Brussels – November 15

- What
- New Frequency
- Properties
- Algorithm
- Worst Case
- Further Work

What...?

Finding frequent items in a continuous stream of items

a b b a a c d e e b c d a a b a b a a c d b a b a a c a d a a

↑timestamp t=1

→ New Frequency Measure: Max-Frequency

→ Incremental Algorithm

→ Worst-Case Analysis

New Frequency: Example

Timestamp 6
Target item a

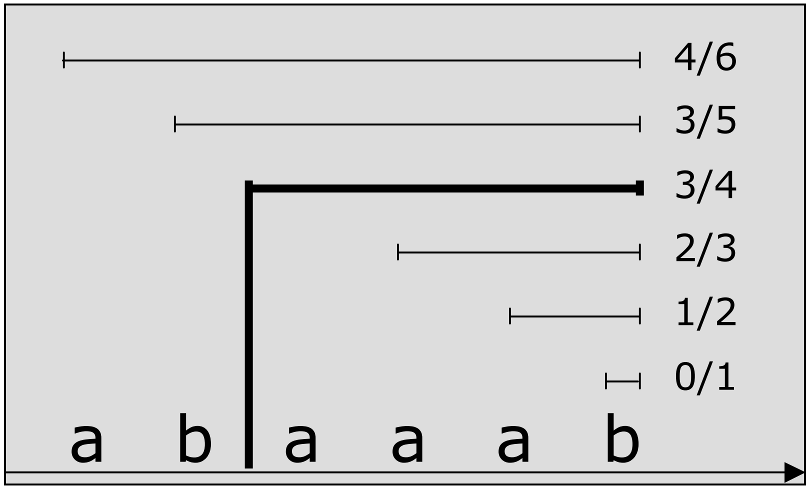
Stream a b a a a b

Time 1 Time 6

$$\mathrm{mfreq}(a, \mathrm{abaaab}) = \max_{k=1..6}(\mathrm{freq}(a, \mathrm{last}(k, \mathrm{abaaab})))$$

$$= \max(0/1, 1/2, 2/3, 3/4, 3/5, 4/6)$$

$$= 3/4$$



New Frequency: Definition

For each item, we consider the window in which it has the highest probability:

Max-Frequency:

$$\mathrm{mfreq}(i, S) := \max_{k=1..|S|}(\mathrm{freq}(i, \mathrm{last}(k, S)))$$

Properties

Checking all possible windows to find the maximal one: **infeasible**

BUT: not every point needs to be checked

↓

Only some special points = the borders

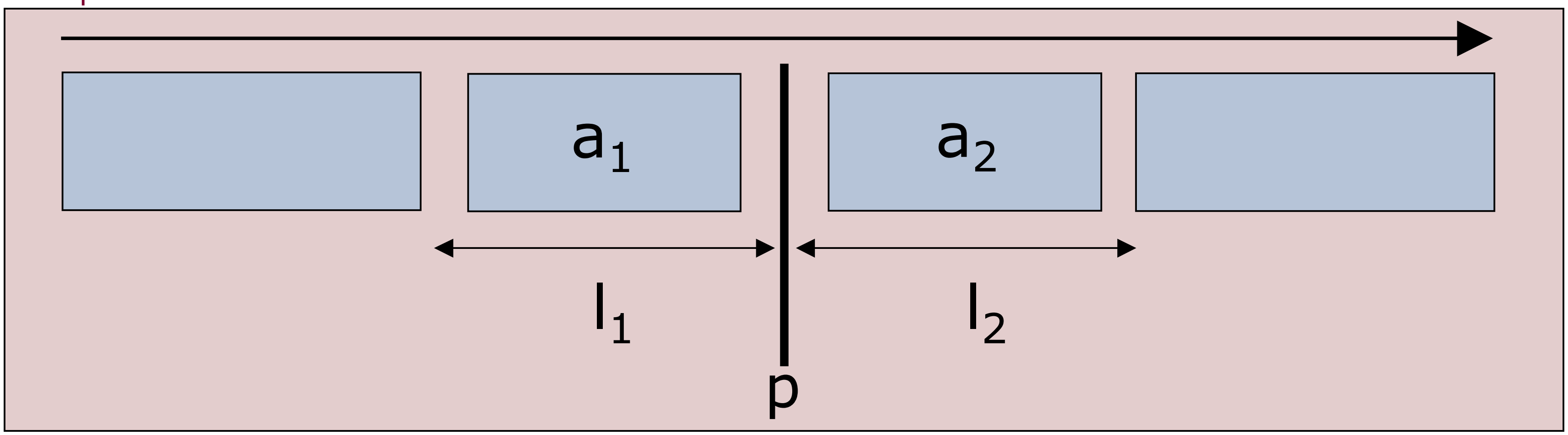
|a a a b b b a b b a b a b a b a b b b b|a a b a b b|a

1	21	27
8/20	**3/6**	**1/1**

How to find the borders?

Target item a

a_i = # occurrences of a in that block



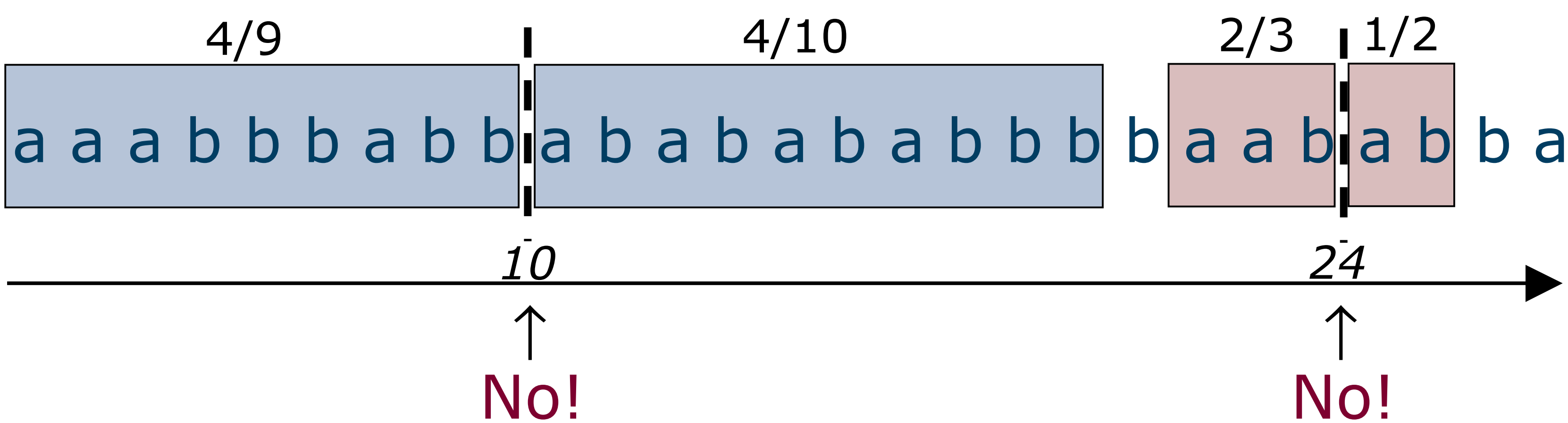
If $a_1/l_1 \geq a_2/l_2$, position p is never the border again!

Very powerful pruning criterion!

If a position p is not a border in *S*,
then it neither can be a border in any extension from *S*.

Example

On timestamp 27, we have S_{27}:



The only borders that need to be remembered:

1	21	27
8/20	3/6	1/1

Algorithm

Output: on every timestamp t: Summary(S_t)

Time t

p_1	…	p_r
x_1/y_1		x_r/y_r

$p_1 < \ldots < p_r$

$x_1/y_1 < \ldots < \boxed{x_r/y_r}$

=

Most recent

= the largest

= the current max-freq

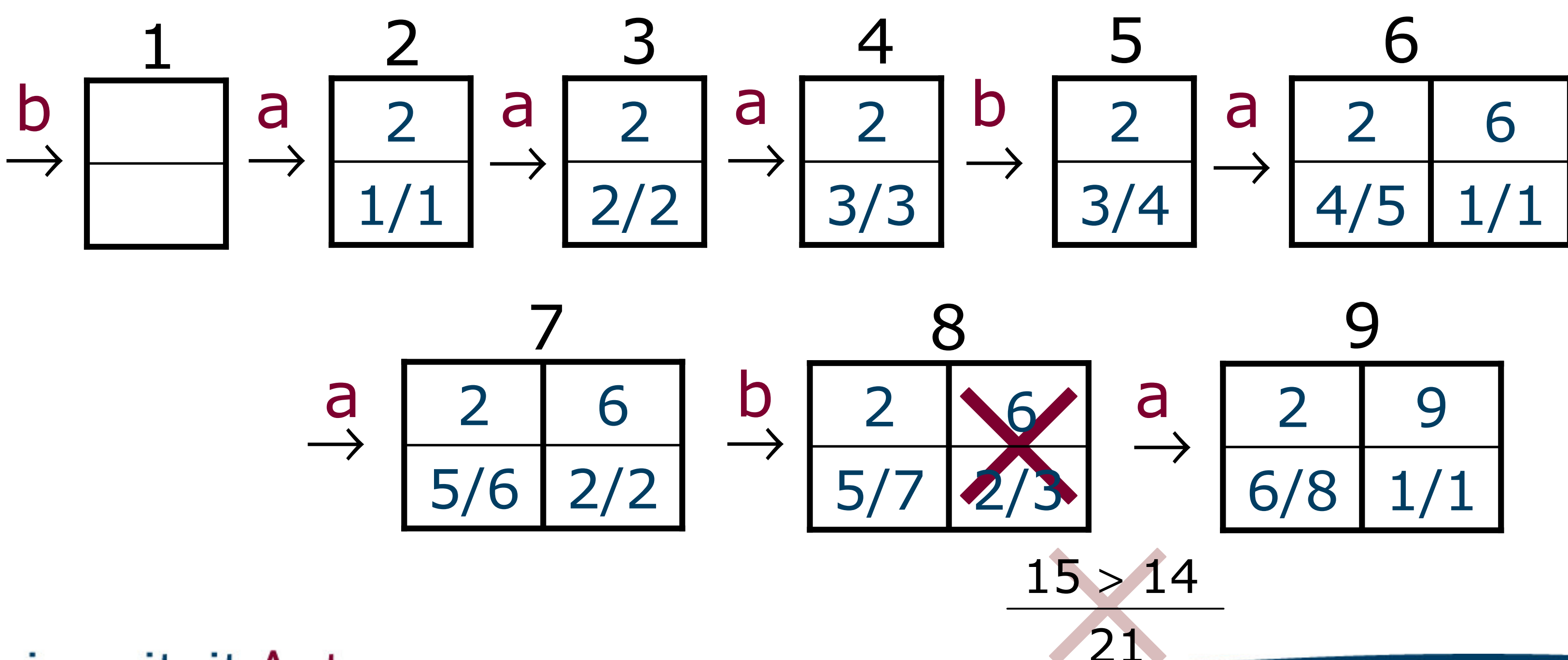
How: on every timestamp, the algo adjusts the stored values based on the newly entered item

- What
- New Frequency
- Properties
- Algorithm
- Worst Case
- Further Work

Example

b | a a a b | a a b | a

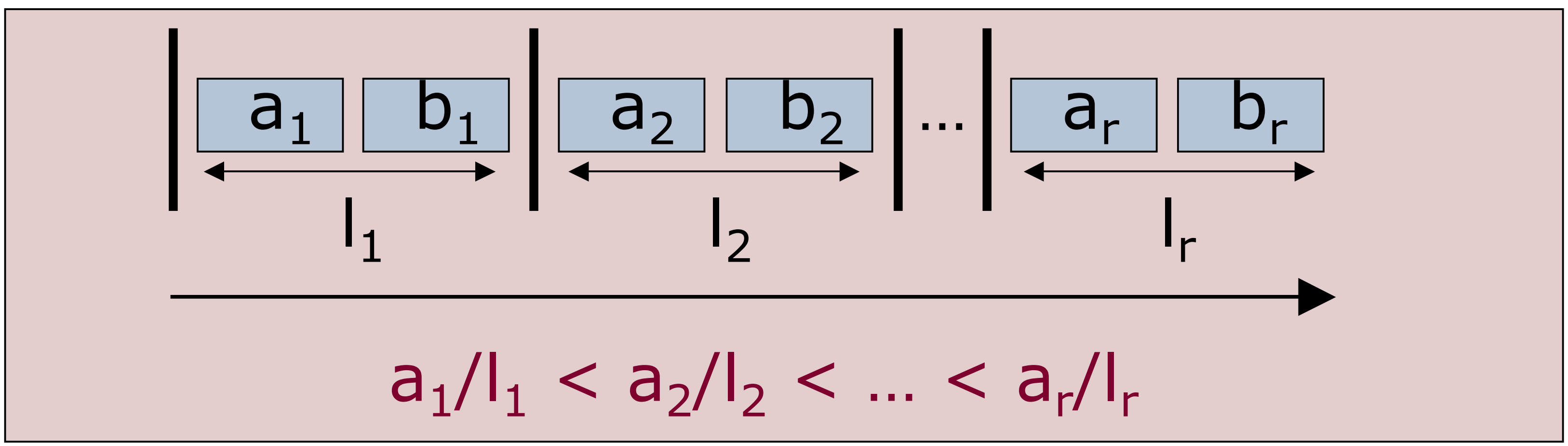
Target item = a



Worst-Case Analysis

For a specific streamlength *l*, we will identify a stream of length *l* that maximizes the number of borders: the Farey stream.

The idea is to have as many blocks as possible, causing as many borders as possible



What Farey has to do with it

$$a_1/l_1 < a_2/l_2 < \dots < a_r/l_r$$

The challenge is for each streamlength $k = l_1 + l_2 + \dots + l_r$ to find such an increasing array of fractions

Solution: Farey sequences

$F_1 = 1/1$
$F_2 = 1/2, 1/1$
$F_3 = 1/3, 1/2, 2/3, 1/1$
$F_4 = 1/4, 1/3, 1/2, 2/3, 3/4, 1/1$

Farey Streams

The Farey Sequence F_n introduces the Farey Stream S_n.

F_5:

1/5 < 1/4 < 1/3 < 2/5 < 1/2 < 3/5 < 2/3 < 3/4 < 4/5 < 1/1

S_5:

|abbbb|abbb|abb|aabbb|ab|aaabb|aab|aaab|aaaab|a

Most Important Result

Theorem:

For streams of length L,

the maximal number of borders is given by N:

$$N = \left(\frac{\pi^2 L}{2}\right)^{2/3} \frac{3}{\pi^2}$$

Remark:

Experiments show that the worst case never happens!

Further Work

- Minimum Window Length
- Focus on multiple targets in the stream

- Make the extension to itemset mining